Magnetic Field Influence on Coulomb Relaxation of Anisotropic Electron-Ion-Plasmas

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The relaxation of velocity-space-anisotropies due to Coulomb collisions is studied within the class of bi-Maxwellian distributions in the presence of an external homogeneous magnetic field, both analytically and numerically. It is found that the anisotropy dependence of the energy exchange collision frequency well separates from its magnetic field dependence. In a certain region of magnetic field strength, the latter can be approximated by a substitution of the thermal electron's Larmor-radius for the Debye-length in the Coulomb-logarithm, as has been conjectured earlier.

1. Introduction

Recently [1], it has been shown that the relaxation of anisotropic bulk distributions due to Coulomb-collisions can be successfully modelled by bi-Maxwellians. This is especially true for distributions distorted, e.g. via wave-heating, perpendicular to a background magnetic field, in which case the energy relaxation, i.e. the relaxation of the perpendicular and parallel temperatures $T_\perp$ and $T_\parallel$, respectively, turned out to be rather insensitive to changes in the Coulomb-collision-operator and/or deviations from bi-Maxwellian shape. For parallel distortions, when $T_\parallel > T_\perp$, the bi-Maxwellian model is somewhat less accurate, and the relaxation appears to be dependent on details of the collision-operator. Nevertheless, using an appropriate nonlinear operator the bi-Maxwellian assumption still provides a useful scheme, giving rise to an upper bound for the relaxation frequency of the anisotropy which is lower than the one obtained, for instance, by a test particle approach.

The magnetic field dependence of the collision operator, however, has been ignored in [1], and it is mainly this dependence which we address in the present paper.

There exists already a long list of publications dealing with Coulomb collisions in magnetized plasmas. Important contributions have been made by Rostoker [2], Schram [3], Montgomery et al. [4], Øien [5], Matsuda [6], and more recently by Ghendrih [7] and Hassan [8, 9], to mention a few of them. As a rule, however, quantitative results are still rare, since only few papers are dealing with a numerical evaluation of the rather intricate formulas.

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2. The Kinetic Equation

We consider a simple, quasineutral electron-ion-plasma in a stationary, homogeneous external magnetic field. The state of the plasma may be initially close to a homogeneous bi-Maxwellian, and it is henceforth assumed that the time evolution takes place within this special class of one-particle-distributions:

$$f_\alpha(v_\perp, v_\parallel, t) = (m_\alpha/2\pi)^{3/2}\, T_{\alpha\parallel}^{-1/2}\, T_{\alpha\perp}^{-1} \exp\left(-\frac{m_\alpha v_\parallel^2}{2T_{\alpha\parallel}} - \frac{m_\alpha v_\perp^2}{2T_{\alpha\perp}}\right), \tag{1}$$

In (1), $\alpha = \mathrm{e, i}$ for electrons and ions, respectively, m_α stands for the particle mass, and $T_{\alpha\parallel}$, $T_{\alpha\perp}$ are the time-dependent temperatures of species α parallel and perpendicular to the magnetic field,

$$T_{\alpha\parallel}(t) := m_\alpha \int \mathrm{d}^3v\, v_\parallel^2\, f_\alpha(v_\parallel, v_\perp, t),$$
$$T_{\alpha\perp}(t) := \tfrac{1}{2} m_\alpha \int \mathrm{d}^3v\, v_\perp^2\, f_\alpha(v_\parallel, v_\perp, t)\,. \tag{2}$$

Instead of $T_{\alpha\perp}$ and $T_{\alpha\parallel}$ we can as well characterize the state by the anisotropy parameter ε_α and the mean temperature T_α, defined by

$$\varepsilon_\alpha := T_{\alpha\perp}/T_{\alpha\parallel}\,, \quad T_\alpha := \tfrac{1}{3} T_{\alpha\parallel} + \tfrac{2}{3} T_{\alpha\perp}\,. \tag{3}$$

Notice that $3 n_\alpha T_\alpha$ is the total kinetic energy density of species α (n_α: particle density of species α).

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The time evolution of (1) is now governed by a generalized Landau-collision-operator, given by

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} f_\alpha\right)_{\mathrm{col}} = 4\pi \sum_\beta R_{\alpha\beta}\, \partial_{v_\alpha} \cdot \int \mathrm{d}^3 v_\beta \int \mathrm{d}^3 k \tag{4}$$

$$\cdot \int_{-\infty}^{0} \mathrm{d}s\, \frac{\boldsymbol{k}}{k^4}\, e^{i(u_\parallel k_\parallel s + Z_{\alpha\beta})} \left(\frac{O_\alpha}{m_\alpha} - \frac{O_\beta}{m_\beta}\right) f_\alpha(\boldsymbol{v}_\alpha)\, f_\beta(\boldsymbol{v}_\beta),$$

where

$$O_\alpha = k_\parallel\, \partial_{v_{\alpha\parallel}} + k_\perp \cos(\phi - \chi + \Omega_\alpha s)\, \partial_{v_{\alpha\perp}},$$

$$O_\beta = k_\parallel\, \partial_{v_{\beta\parallel}} + k_\perp \cos(\psi - \chi + \Omega_\beta s)\, \partial_{v_{\beta\perp}}$$

and $v_{\alpha\perp}, v_{\alpha\parallel}, \phi; v_{\beta\perp}, v_{\beta\parallel}, \psi; k_\perp, k_\parallel, \chi$ are the cylindrical coordinates of $\boldsymbol{v}_\alpha, \boldsymbol{v}_\beta, \boldsymbol{k}$, respectively. the external magnetic field $\boldsymbol{B}$ is pointing in the z-direction. in (4) the following definitions have been used:

$$Z_{\alpha\beta} = \frac{k_\perp v_{\alpha\perp}}{\Omega_\alpha}\,[\sin(\phi - \chi) - \sin(\phi - \chi + \Omega_\alpha s)]$$

$$- \frac{k_\perp v_{\beta\perp}}{\Omega_\beta}\,[\sin(\psi - \chi) - \sin(\psi - \chi + \Omega_\beta s)],$$

$$R_{\alpha\beta} = \frac{z_\alpha^2 z_\beta^2 n_\beta}{m_\alpha}\, \frac{e^4}{32\pi^4 \varepsilon_0^2}, \quad u_\parallel = v_{\beta\parallel} - v_{\alpha\parallel},$$

$$\Omega_\alpha = \frac{z_\alpha}{m_\alpha} e B.$$

Ω_α denotes the Larmor-frequency of species α.

In the derivation of (4), which can be found e.g. in [4], [8], or [10], a number of assumptions enters, such as: weak coupling limit, nonrelativistic particle motion, classical Coulomb-interaction and -statistics or binary collision approximation. The standard requirements to insure validity of this regime are:

i) the plasma parameter $g = (n\lambda_\mathrm{D}^3)^{-1}$, where λ_D is the (electron-)Debye-length, is sufficiently small, and

ii) the Larmor-radius for electrons with thermal velocity exceeds the Landau-length λ_0 (impact parameter for 90°-collision of electrons with thermal velocity), which itself is much larger than the de-Broglie-wavelength of such electrons.

In this weak-interaction-approximation a collision produces only a small perturbation in the orbit of a particle, implying that the procedure of integration along unperturbed orbits, resulting in a Landau-type collision operator, remains valid (for details, see e.g. [7]). (Note that these assumptions are well satisfied in present day tokamak experiments.) In addition, collective (mean-)field effects as well as an external wave energy- or particle-source-term are neglected and, finally, it should be mentioned that in deriving (4) gyrotropic distributions have been assumed.

Furthermore, as is well known, (4) conserves the total energy, momentum, and particle number. It is equivalent to the Rostoker-equation [2], as can be seen by expanding the term $\exp(i Z_{\alpha\beta})$ into two infinite Bessel-series. Equation (4) reduces to the ordinary Landau-equation [11] in the limit of $B \to 0$.

3. Evaluation of Anisotropy Relaxation

With (4) we can calculate $\dot{T}_{\alpha\perp}$ and $\dot{T}_{\alpha\parallel}$:

$$\dot{T}_{\alpha\parallel} = m_\alpha \int \mathrm{d}^3 v\, v_\parallel^2 \frac{\mathrm{d}}{\mathrm{d}t} f_\alpha(v_\parallel, v_\perp, t),$$

$$\dot{T}_{\alpha\perp} = \tfrac{1}{2} m_\alpha \int \mathrm{d}^3 v\, v_\perp^2 \frac{\mathrm{d}}{\mathrm{d}t} f_\alpha(v_\parallel, v_\perp, t)$$

by inserting (4) and (1). The tedious but rather straightforward algebra is skipped here, for details see [10]. The result already obtained in [9] reads

$$\dot{T}_{\alpha\parallel} = 8\pi \sum_\beta R_{\alpha\beta} \int \mathrm{d}^3 k\, \frac{k_\parallel^2}{k^4} \int_0^\infty \mathrm{d}s\, e^{-S_\alpha - S_\beta}$$

$$\cdot \left[1 - k_\parallel^2 s^2 T_{\alpha\parallel}\left(\frac{1}{m_\alpha} + \frac{1}{m_\beta}\right) - k_\perp^2 s T_{\alpha\parallel}\left(\frac{\sin \Omega_\alpha s}{m_\alpha \Omega_\alpha} + \frac{\sin \Omega_\beta s}{m_\beta \Omega_\beta}\right)\right], \tag{5a}$$

$$\dot{T}_{\alpha\perp} = -4\pi \sum_\beta R_{\alpha\beta} \int \mathrm{d}^3 k\, \frac{k_\perp^2}{k^4} \int_0^\infty \mathrm{d}s\, e^{-S_\alpha - S_\beta}$$

$$\cdot \left\{\frac{k_\perp^2 \sin \Omega_\beta s}{m_\beta \Omega_\beta}(T_{\alpha\perp} - T_{\beta\perp}) + k_\parallel^2 s\left[\frac{T_{\alpha\perp}}{m_\alpha} + \frac{T_{\alpha\perp}}{m_\beta} - \frac{T_{\alpha\parallel}}{m_\alpha} - \frac{T_{\beta\parallel}}{m_\beta}\right]\right\} \frac{\sin \Omega_\alpha s}{\Omega_\alpha} \tag{5b}$$

with the following abbreviations:

$$S_\alpha = 2\frac{T_{\alpha\perp}}{m_\alpha \Omega_\alpha^2} k_\perp^2 \sin^2(\tfrac{1}{2}\Omega_\alpha s) + \tfrac{1}{2}\frac{T_{\alpha\parallel}}{m_\alpha} k_\parallel^2 s^2,$$

$$S_\beta = 2\frac{T_{\beta\perp}}{m_\beta \Omega_\beta^2} k_\perp^2 \sin^2(\tfrac{1}{2}\Omega_\beta s) + \tfrac{1}{2}\frac{T_{\beta\parallel}}{m_\beta} k_\parallel^2 s^2.$$

The k-integral has to be cut off for large $|\boldsymbol{k}|$ because of the well-known logarithmic divergence. We find it convenient to use cylindrical coordinates and to cut off only the $k_\perp$-integral to remove the singularity. One then obtains the famous Coulomb-logarithm in the case of $B=0$.

We are now going to evaluate (5) for special initial conditions, namely $T_e = T_i$ and either $\varepsilon_e \neq 1$, $\varepsilon_i = 1$ or $\varepsilon_i \neq 1$, $\varepsilon_e = 1$ for $t=0$. That is the special case of only one species being anisotropic and the other one being isotropic and having the same mean temperature. It is further assumed that $T_e = T_i$ is valid for all times during relaxation.

Thus the only time dependent variable is the anisotropy parameter ε_α of the anisotropic species, which evolves in time according to (3) as

$$\dot{\varepsilon}_\alpha = T_{\alpha\|}^{-1} (\dot{T}_{\alpha\perp} - \varepsilon_\alpha \dot{T}_{\alpha\|}) . \tag{6}$$

The contributions to the time evolution of ε_α can be splitted into a like- and unlike-particle-collision-term.

The Self-Contribution-Term

It is obtained by taking only the summand $\beta = \alpha$ in (5) and cutting off the $k_\perp$-integral at a lower and an upper boundary of $k_D = \lambda_D^{-1}$ and $k_0 = \lambda_0^{-1}$, respectively, where λ_D and λ_0 are the Debye- and Landau-length, respectively.

The result is:

$$\dot{\varepsilon}_\alpha|_{\text{self}}(B, \varepsilon_\alpha) = 4\,\nu_\alpha \frac{(2\varepsilon_\alpha+1)^{5/2}(1-\varepsilon_\alpha)}{\varepsilon_\alpha} \int\limits_{\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{6\varepsilon_\alpha}{1+2\varepsilon_\alpha}}^{\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{6\varepsilon_\alpha}{1+2\varepsilon_\alpha}\Lambda^2} \frac{dx}{x} F(x, \varepsilon_\alpha),$$

$$\dot{T}_\alpha|_{\text{self}} = 0, \tag{7}$$

where

$$F(x, \varepsilon) = \int_0^\infty dt\, e^{-2x \sin^2 t \sqrt{\frac{\varepsilon}{2x}}} \int_0^\infty dy\, y^2 (y^2-1)\, e^{-y^2-2yt} \tag{8}$$

is a monotonically increasing function of x reaching its asymptotic value $F(\infty, \varepsilon)$ for $x \gtrsim \varepsilon$ in good approximation,

$$\omega_{p\alpha} = \sqrt{\frac{Z_\alpha^2 n_\alpha}{m_\alpha} \frac{e^2}{\varepsilon_0}}, \quad \Lambda := \frac{\lambda_D}{\lambda_0} = 12\pi g \quad \text{and}$$

$$\nu_\alpha = \frac{n_\alpha Z_\alpha^4}{\sqrt{m_\alpha}\, T_\alpha^{3/2}} \frac{e^4}{24\sqrt{3}\,\pi^{3/2}\varepsilon_0^2} .$$

$\omega_{p\alpha}$ is the plasmafrequency of species α.

In the limit $B \to 0$, corresponding to $x_{\min} \to \infty$, where $x_{\min}$ is the lower boundary of the x-integral in (7), $F(x \to \infty, \varepsilon)$ yields

$$F(x \to \infty, \varepsilon) = \int_0^\infty dt\, e^{-\varepsilon t^2} \int_0^\infty dy\, y^2 (y^2-1)\, e^{-y^2-2yt}$$
$$= \tfrac{1}{2}\varepsilon \frac{d}{d\varepsilon} \varepsilon \frac{d}{d\varepsilon} g(\varepsilon), \tag{9}$$

where

$$g(\varepsilon) = \frac{\operatorname{artanh}\sqrt{1-\varepsilon}}{\sqrt{1-\varepsilon}} \quad \text{for } \varepsilon < 1, \quad g(\varepsilon) = 1 \text{ for } \varepsilon = 1,$$

and

$$g(\varepsilon) = \frac{\arctan\sqrt{\varepsilon-1}}{\sqrt{\varepsilon-1}} \quad \text{for } \varepsilon > 1,$$

and one thus obtains

$$\dot{\varepsilon}_\alpha|_{\text{self}}(B=0, \varepsilon_\alpha) = \nu_\alpha \ln\Lambda \frac{(1+2\varepsilon_\alpha)^{5/2}[(2+\varepsilon_\alpha)\, g(\varepsilon_\alpha) - 3]}{1-\varepsilon_\alpha} \tag{10}$$

in accordance with [1].

On the other hand, the limit $B \to \infty$ can be evaluated by considering $F(x \to 0, \varepsilon_\alpha)$, which yields

$$F(x \to 0, \varepsilon_\alpha) = x^2/2\varepsilon_\alpha + O(x^3)$$

and thus

$$\dot{\varepsilon}_\alpha|_{\text{self}}(B \to \infty, \varepsilon_\alpha) \tag{11}$$
$$\to \quad \nu_\alpha\, 36 \sqrt{1+\varepsilon_\alpha}\,(1-\varepsilon_\alpha)(\Lambda^4 - 1) \frac{\omega_{p\alpha}^4}{\Omega_\alpha^4} + O(\Omega_\alpha^{-6}).$$

It should be noted that the result for this limit depends on whether one cuts off the k-integral spherically or cylindrically; $\dot{\varepsilon}$, nevertheless, vanishes as $B \to \infty$. However, as pointed out, the chosen collision operator may not be valid in this region of an ultra-strong magnetic field ($\omega_{p\alpha}/\Omega_\alpha \lesssim \Lambda^{-1}$) [12].

Equation (7) has been evaluated numerically; the result is presented in Figure 1, showing the collision frequency, normalized by its $B = 0$ value, as a function of the inverse magnetic field strength ($\propto \Omega_\alpha^{-1}$), and of the anisotropy-parameter ε.

Concerning the ε-dependence, it should be emphasized that the relative collision frequency is nearly ε-independent in the considered region of anistropy. This implies that one can separate the influence of magnetic field and anisotropy, appearing as two independent factors in the collision frequency, namely

$$\dot{\varepsilon}_\alpha(\Omega_\alpha, \varepsilon_\alpha) \cong \dot{\varepsilon}_\alpha(0, \varepsilon_\alpha)\, \gamma_\alpha(\Omega_\alpha) \tag{12}$$

with $\dot{\varepsilon}_\alpha(0, \varepsilon_\alpha)$ given by (10) and $\gamma_\alpha(\Omega_\alpha)$ defined conveniently as

$$\gamma_\alpha(\Omega_\alpha) := \dot{\varepsilon}_\alpha(\Omega_\alpha, 1)/\dot{\varepsilon}_\alpha(0, 1) \tag{13}$$

(the quotient is to be understood as the limit of $\varepsilon_\alpha \to 1$).

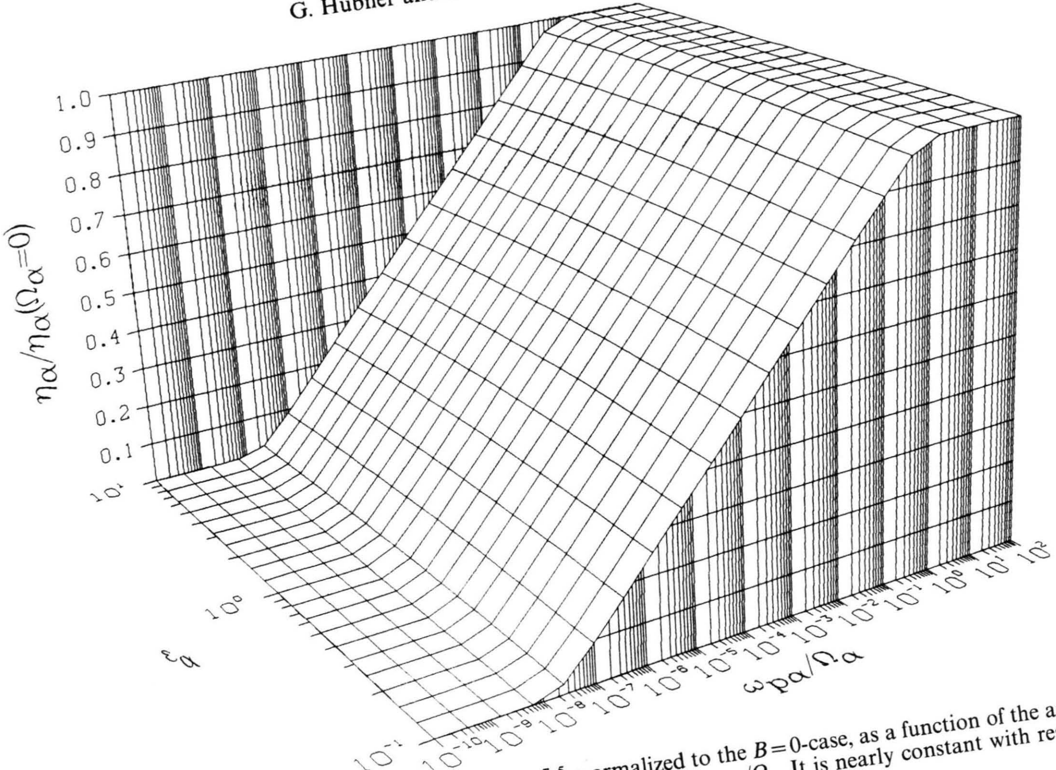

Fig. 1. The relative collision frequency for $\Lambda=10^{7.5}$, normalized to the $B=0$-case, as a function of the anisotropy parameter, ε_α, and of the ratio of plasma- to Larmor-frequency of species α, $\omega_{p\alpha}/\Omega_\alpha$. It is nearly constant with respect to ε_α, and three characteristic regions exist with distinct magnetic field dependency.

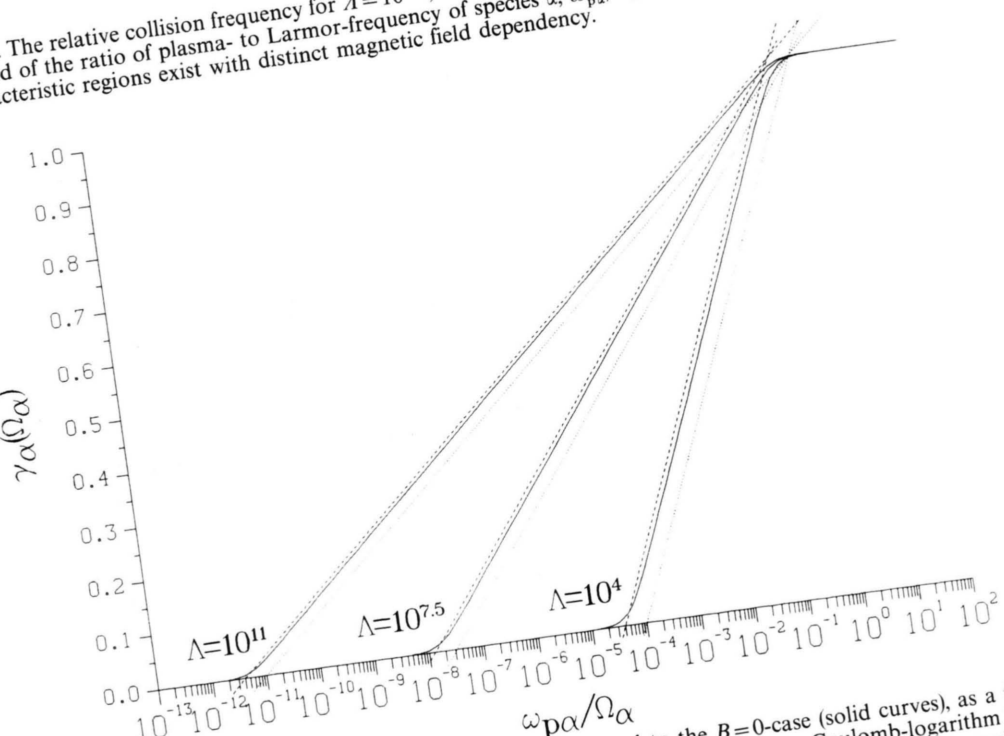

Fig. 2. The relative collision frequency for $\varepsilon_\alpha=1$, normalized to the $B=0$-case (solid curves), as a function of the ratio of plasmafrequency to Larmor-frequency of species α, shown for three values of the Coulomb-logarithm $\ln \Lambda$. In an intermediate region of magnetic field strengths, it can be well approximated by straight lines (dashed lines: approximation of Montgomery et al.; dotted lines: simple approximation).

The collision frequency η_α is defined as usually:

$$\dot{\varepsilon}_\alpha(\Omega_\alpha, \varepsilon_\alpha) = \eta_\alpha(\Omega_\alpha, \varepsilon_\alpha)\,(1-\varepsilon_\alpha)\,.$$

The numerical evaluation of (13) with (7) and (10) for three different values of the Coulomb-logarithm yields the solid curves of Figure 2. One can recognize three characteristic regions of the magnetic field strength:

i) the quasi-magnetic-field-free-region, where $\omega_{\mathrm{p}\alpha}/\Omega_\alpha \gtrsim 1$. In this region the collision process is scarcely influenced by the gyro-motion of the particles. The integration over particle orbits in the collision operator can be well approximated by an integration over the straight line $B=0$-trajectories.

ii) the intermediate region, where $\Lambda^{-1} < \omega_{\mathrm{p}\alpha}/\Omega_\alpha \lesssim 1$. Here, one finds a logarithmic decrease of the collision frequency with increasing magnetic field, which can be interpreted (see below) as an effective reduction of the maximum interaction range perpendicular to the external field. It turns out that for like-particle-collisions no energy exchange via collision occurs if the impact parameter is greater than the order of a typical gyro-radius.

iii) the ultra-strong-magnetic-field-region, where $\omega_{\mathrm{p}\alpha}/\Omega_\alpha \lesssim \Lambda^{-1}$. In this region, the weak-interaction-assumption breaks down, because the effective collisions have now impact parameters smaller than the Landau-length λ_0 [12]. Nevertheless, the collision-frequency approaches zero monotonically, as can also be verified by recognizing that the collisions are nearly one-dimensional, with no energy-exchange at all (at least between the parallel and perpendicular temperatures of like particles). It is the conservation of magnetic moment which in the $B\to\infty$-limit prevents an energy exchange between parallel and perpendicular degrees of freedom.

In the intermediate region, one can make a simple approximation to γ_α (dotted curves in Fig. 2):

$$\gamma_\alpha(\Omega_\alpha) \cong 1 + \frac{\lg \omega_{\mathrm{p}\alpha}/\Omega_\alpha}{\lg \Lambda}\,. \tag{14}$$

This is equivalent to a substitution of the Debye-length λ_{D} in $\ln \Lambda$ of (10) by the gyro-radius of a species-α-particle with thermal velocity, a point which has been conjectured earlier by several authors, e.g. by Salat and Joyce [13] (see also the note added in proof of [14]).

A more subtle approximation is obtained if one substitutes λ_{D} in $\ln \Lambda$ of the Landau-collision-operator for the $B=0$-case (given e.g. in [1]) by the gyro-radius of a species-α-particle with the relative velocity (which one has to integrate over) of two colliding species-α-particles, giving the dashed curves in Figure 2. This procedure is due to Montgomery et al. [14] and, as Fig. 2 shows, yields a still better approximation in this region of magnetic field. It also provides the feature of nearly separating the anisotropy- and magnetic-field-influence on the relaxation frequency. For details about the calculation of the Montgomery-approximation see [10]; the result is:

$$\gamma_{\alpha,m}(\Omega_\alpha) \cong 1 + \frac{\lg \omega_{\mathrm{p}\alpha}/\Omega_\alpha}{\lg \Lambda} + (1-\gamma+\ln 4)/2\ln\Lambda\,, \tag{15}$$

where $\gamma = 0.5772156\ldots$ is Euler's constant.

The Ion Contribution to Electron-Anisotropy-Relaxation

Taking $\alpha=\mathrm{e}$, $\beta=\mathrm{i}$ in (5) and retaining only terms of first non-vanishing order in $m_{\mathrm{e}}/m_{\mathrm{i}} =: \mu$, we get

$$\dot{\varepsilon}_{\mathrm{e}}|_{\mathrm{ion}} = \boxed{\sqrt{2}Z_{\mathrm{i}}}\; 4\nu_{\mathrm{e}}\,\frac{(2\varepsilon+1)^{5/2}(1-\varepsilon)}{\varepsilon} \int\limits_{\frac{\omega_{\mathrm{pe}}^2}{\boxed{2}\Omega_{\mathrm{e}}^2}\frac{6\varepsilon}{1+2\varepsilon}}^{\frac{\omega_{\mathrm{pe}}^2}{\boxed{2}\Omega_{\mathrm{e}}^2}\frac{6\varepsilon}{1+2\varepsilon}\Lambda^2} \frac{\mathrm{d}x}{x}\,F(x,\varepsilon_\alpha) + \boxed{O(\mu)}\,, \qquad \dot{T}_{\mathrm{e}}|_{\mathrm{ion}} = \boxed{O(\mu)}\,. \tag{16}$$

This differs from (7) only at the bordered places. Notice, that $\dot{T}_{\mathrm{e}}|_{\mathrm{ion}}$, though small, is not exactly zero, due to the restriction imposed on the relaxation process. The result in (16) is only valid for not too strong magnetic field strengths, namely $\omega_{\mathrm{pe}}/\Omega_{\mathrm{e}} \gtrsim \mu := m_{\mathrm{e}}/m_{\mathrm{i}}$. In this case, one can regard the ions as heavy particles, which are hardly influenced by gyro-motion during collisions. The limit $B\to 0$ is readily obtained from (16):

$$\dot{\varepsilon}_{\mathrm{e}}|_{\mathrm{ion}}(B=0) = \sqrt{2}\,Z_{\mathrm{i}}\,\dot{\varepsilon}_{\mathrm{e}}|_{\mathrm{self}} + O(\mu),$$
$$\dot{T}_{\mathrm{e}}|_{\mathrm{ion}}(B=0) = O(\mu) \tag{17}$$

in accordance with [1].

The main result is that the effect of the ions on electron anisotropy relaxation takes nearly the same form with or without an external magnetic field, and is by a factor of order unity greater than the self-interaction contribution.

The Electron Contribution to Ion-Anisotropy-Relaxation

Taking now $\alpha = \mathrm{i}$, $\beta = \mathrm{e}$ in (5), we obtain to first order in $\mu = m_\mathrm{e}/m_\mathrm{i}$

$$\dot{\varepsilon}_\mathrm{i}|_\mathrm{el} = \frac{6\sqrt{6}Z_\mathrm{i}^2}{\sqrt{\pi}} v_\mathrm{e}\mu \frac{1-\varepsilon}{1+2\varepsilon} \int\limits_{\frac{\omega_\mathrm{pe}^2}{\Omega_\mathrm{e}^2}}^{\frac{\omega_\mathrm{pe}^2}{\Omega_\mathrm{e}^2}\Lambda^2} \frac{\mathrm{d}x}{x} \int\limits_0^\infty \mathrm{d}t\, e^{-2x\sin^2 t\sqrt{\frac{1}{2x}}} \cdot \int\limits_0^\infty \mathrm{d}y \frac{1-2\varepsilon y^2}{(1+y^2)^2} e^{-t^2 y^2} + O(\mu^2), \tag{18}$$

$$\dot{T}_\mathrm{i}|_\mathrm{el} = \frac{4\sqrt{6}Z_\mathrm{i}^2 T_\mathrm{e}}{\sqrt{\pi}} v_\mathrm{e}\mu \frac{1-\varepsilon}{1+2\varepsilon} \int\limits_{\frac{\omega_\mathrm{pe}^2}{\Omega_\mathrm{e}^2}}^{\frac{\omega_\mathrm{pe}^2}{\Omega_\mathrm{e}^2}\Lambda^2} \frac{\mathrm{d}x}{x} \int\limits_0^\infty \mathrm{d}t\, e^{-2x\sin^2 t\sqrt{\frac{1}{2x}}} \cdot \int\limits_0^\infty \mathrm{d}y \frac{1-2y^2}{(1+y^2)^2} e^{-t^2 y^2} + O(\mu^2). \tag{19}$$

The change in the mean temperature of the ion-species (19) is now of the same magnitude as the change in anisotropy (18). Because (19) is inconsistent with $T_\mathrm{e}(t) = T_\mathrm{i}(t)$ (though only to order μ), (18) can not be taken to describe the real evaluation of the anisotropy in consistency with this model. Nevertheless, one can make a rough estimation about the self- and the electron-contribution to ion-anisotropy-relaxation: By evaluating (7) for $\alpha = \mathrm{i}$, one recognizes that the contribution of the electrons is about a factor of order $\sqrt{\mu}$ smaller than the self-contribution, which is also true for $B = 0$ [1].

4. Summary and Conclusion

In this work, we have investigated the influence of both, magnetic field and anisotropy on the velocity-space-anisotropy-relaxation of a quasineutral, spatially homogeneous electron-ion-plasma by means of a generalized Landau-kinetic equation. We made the only assumption, that the relaxation process takes place within the class of bi-Maxwell-distributions (which is numerically justified in [1] for $B = 0$), and get an ordinary differential equation for the time development of the anisotropy parameter ε_α of species α, the right-hand side of which can be identified with $\eta_\alpha(\varepsilon_\alpha, \Omega_\alpha) \cdot (1 - \varepsilon_\alpha)$, where η_α and Ω_α are the effective collision-frequency and the Larmor-frequency, respectively. By evaluating η_α analytically and numerically, it turned out that the contribution of the magnetic field and of the anisotropy is well separated, giving rise to a factor γ_α, so that $\eta_\alpha(\varepsilon_\alpha, \Omega_\alpha) = \eta_\alpha(0, \varepsilon_\alpha) \cdot \gamma_\alpha(\Omega_\alpha)$, where γ_α is a monotonically decreasing function of Ω_α.

We have also calculated some approximations of γ_α, the Montgomery-approximation [14], and a simpler one: $\gamma_\alpha \cong 1 + \omega_{\mathrm{p}\alpha}/\Omega_\alpha$, where $\omega_{\mathrm{p}\alpha}$ denotes the plasma-frequency of species α, and we have found that these approximations are rather good ones in an intermediate region of magnetic field strength: $\Lambda^{-1} \lesssim \omega_{\mathrm{p}\alpha}/\Omega_\alpha \lesssim 1$, where $\ln \Lambda$ denotes the Coulomb-logarithm.

It is this region which is now gradually attainable, at least for the electron species, in present day fusion-experiments, especially in tokamaks. A further increase of the magnetic field strength, e.g. by means of superconductive coils, hence, reduces the collision frequency, which may be accompanied by an improved plasma confinement.

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